2020

Elec 301 MINI Project 1

MATHEMATICAL AND COMPUTER TOOLS

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PART 1:

We were given the transfer function $T(s) = 0.125 * \frac{10^5}{s+10^5} * \frac{10^6}{s+10^6} * \frac{10^7}{s+10^7}$

and its corresponding circuit (figure 1.1) as well as the information that C1>C2>C3, 4 of the resistors are 1k resistors and 2 are 2k resistors.

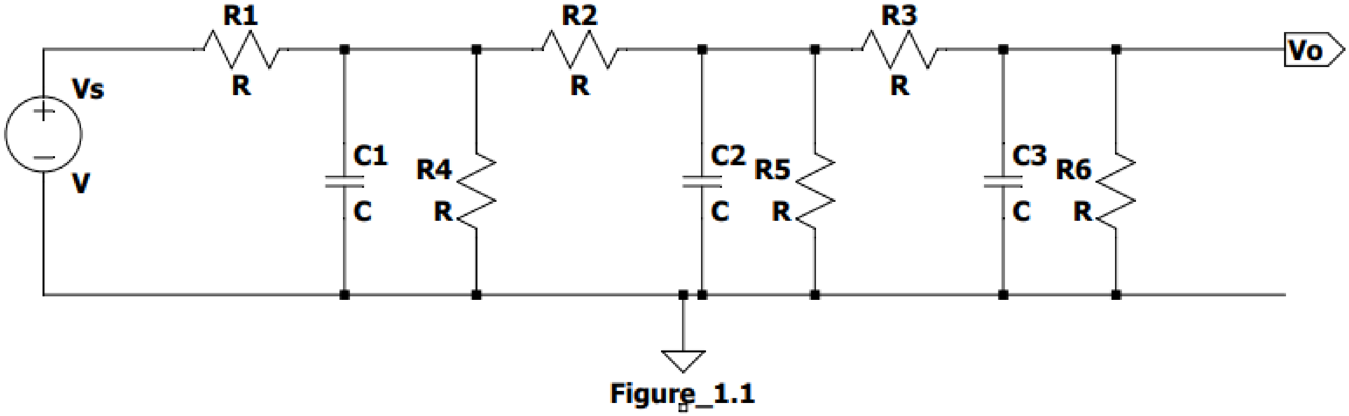

Figure 1.1 The initial circuit given

Knowing that T(s) = A_m * F_L(s) * F_H(s) where A_m is the mid band gain, F_L(s) is the low frequency high pass and F_H(s) is the high frequency low pass.

And recognizing that all the poles of our transfer function happen at high frequencies I approximated the mid band circuit by shorting all the capacitors and by setting all the resistors to 1k

By analyzing this circuit, I found that the voltage gain is lower than our expected mid-band gain. The voltage at Vo was dependent on the current flowing through R6, and I noticed that the current going through R6 is the leftover current that doesn't get siphoned off by the shunt resistors R4 and R5. By increasing the resistance values of R4 and R5 we cause them to siphon less current and thereby increase our voltage gain.

By setting R4 and R5 to 2k and recalculating the gain I found the expected of gain 0.125 for mid-band and confirmed my resistor values.

Knowing the resistor values, I used the method of open circuit short circuit time constants to find C1 = 20nF C2=2nF and C3 = 0.2nF

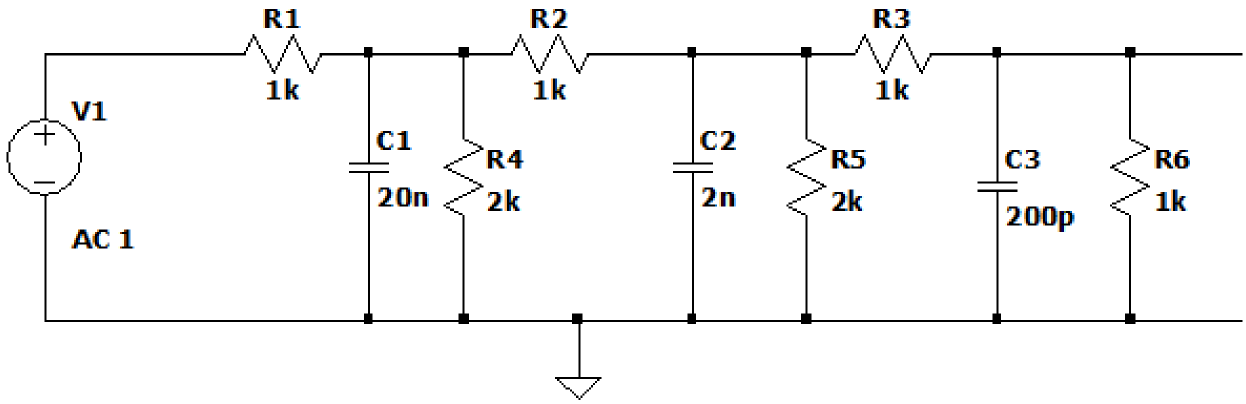

Figure 1.2. The circuit from part 1 after finding R values and C values

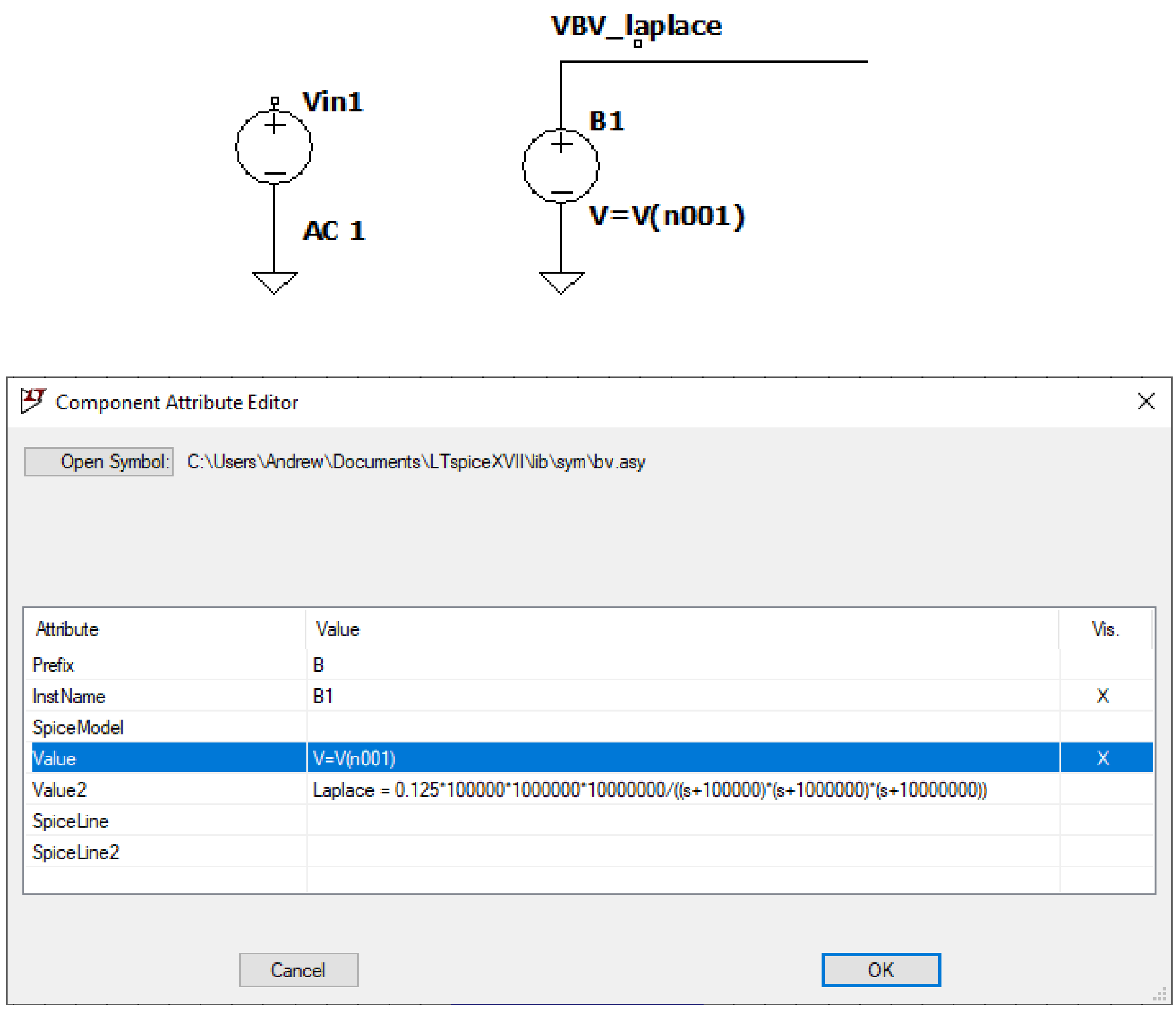

Figure 1.3. The LTspice equivalent of sfxer blocks and their setup configuration for the transfer function given in part 1

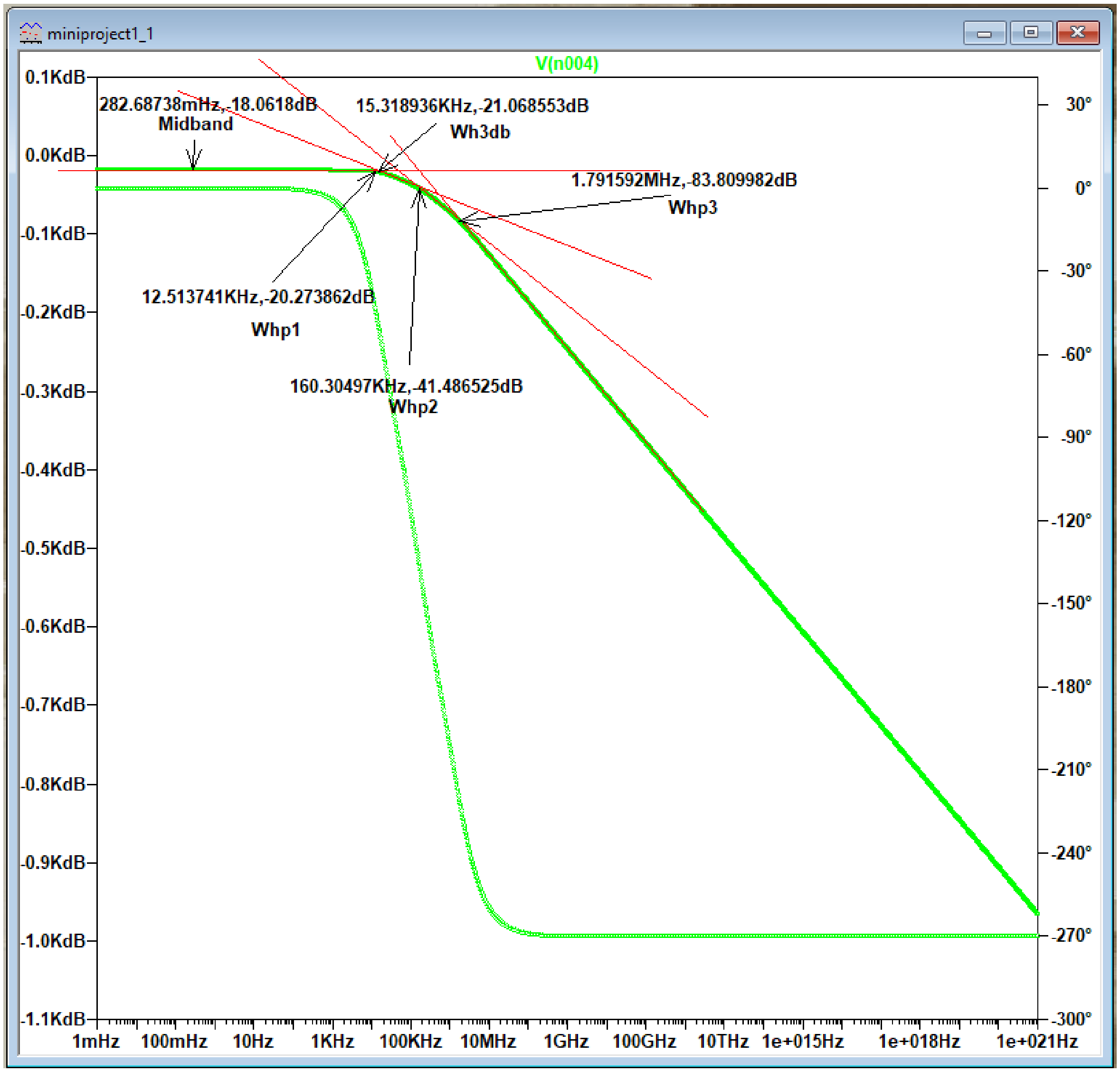

Figure 1.4 Bode plot of the simulated circuit

Graphical analysis of the bode plot of the circuit shows a 3dB point at 15.18 (KHz) or 95.404E3 (1/sec) as well as a negative slope of 60dB/decade. The 3 poles of our transfer function are close enough together that it would be more realistic to treat them as a triple pole rather than 3 individual poles.

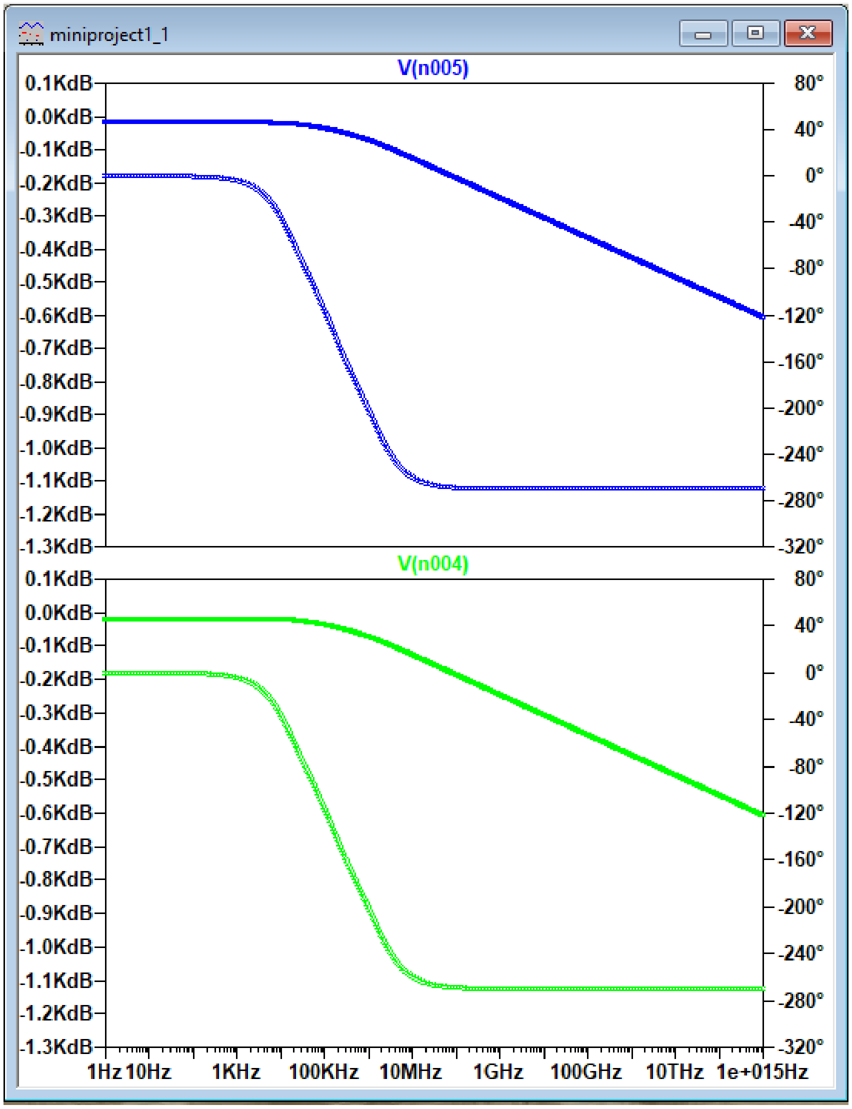

Figure 1.5 The bode plot of my simulated circuit(bottom) and the bode plot of the transfer function block(top)

Graphically comparing the 2 plots found that they are nearly identical and the only difference being a 0.2dB difference in magnitude just after the 3dB point and a 0.4 degree difference in phase angle in the same area.

PART 2:

Supplied with the following circuit

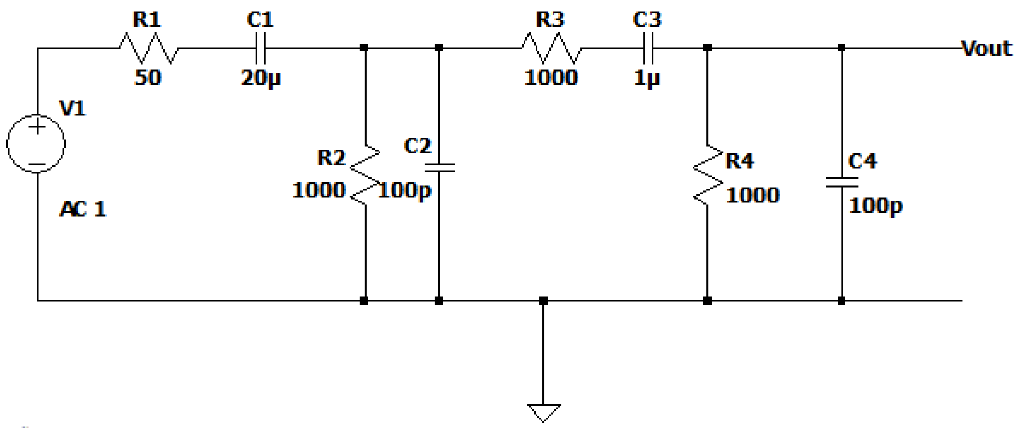

Figure 2.1 The provided circuit to simulate

I used the method of open circuit short circuit time constants to find the values of the poles of the transfer function for each value of C3 in the circuit given in the following table

Table of calculated values:

C3	500nF	1µF	2µF	5µF	10µF
W_{lp1}(1/sec)	47.61	47.61	47.61	47.61	33.33
(Hz)	7.58	7.58	7.58	7.58	5.30
W_{lp2}(1/sec)	976.7	488.37	244.2	97.67	69.75
(Hz)	155.45	77.73	38.87	15.54	11.10
W_{Hp1}(1/sec)	1.95E7	1.95E7	1.95E7	1.95E7	1.95E7
(Hz)	3.11E6	3.11E6	3.11E6	3.11E6	3.11E6
W_{Hp2}(1/sec)	2.2E8	2.2E8	2.2E8	2.2E8	2.2E8
(Hz)	3.50E7	3.50E7	3.50E7	3.50E7	3.50E7
W_{L3dB}(1/sec)	977.86	490.69	248.80	108.66	77.30
(Hz)	155.63	78.09	39.60	17.29	12.30
W_{H3dB}(1/sec)	1.95E7	1.95E7	1.95E7	1.95E7	1.95E7
(Hz)	3.10E6	3.10E6	3.10E6	3.10E6	3.10E6

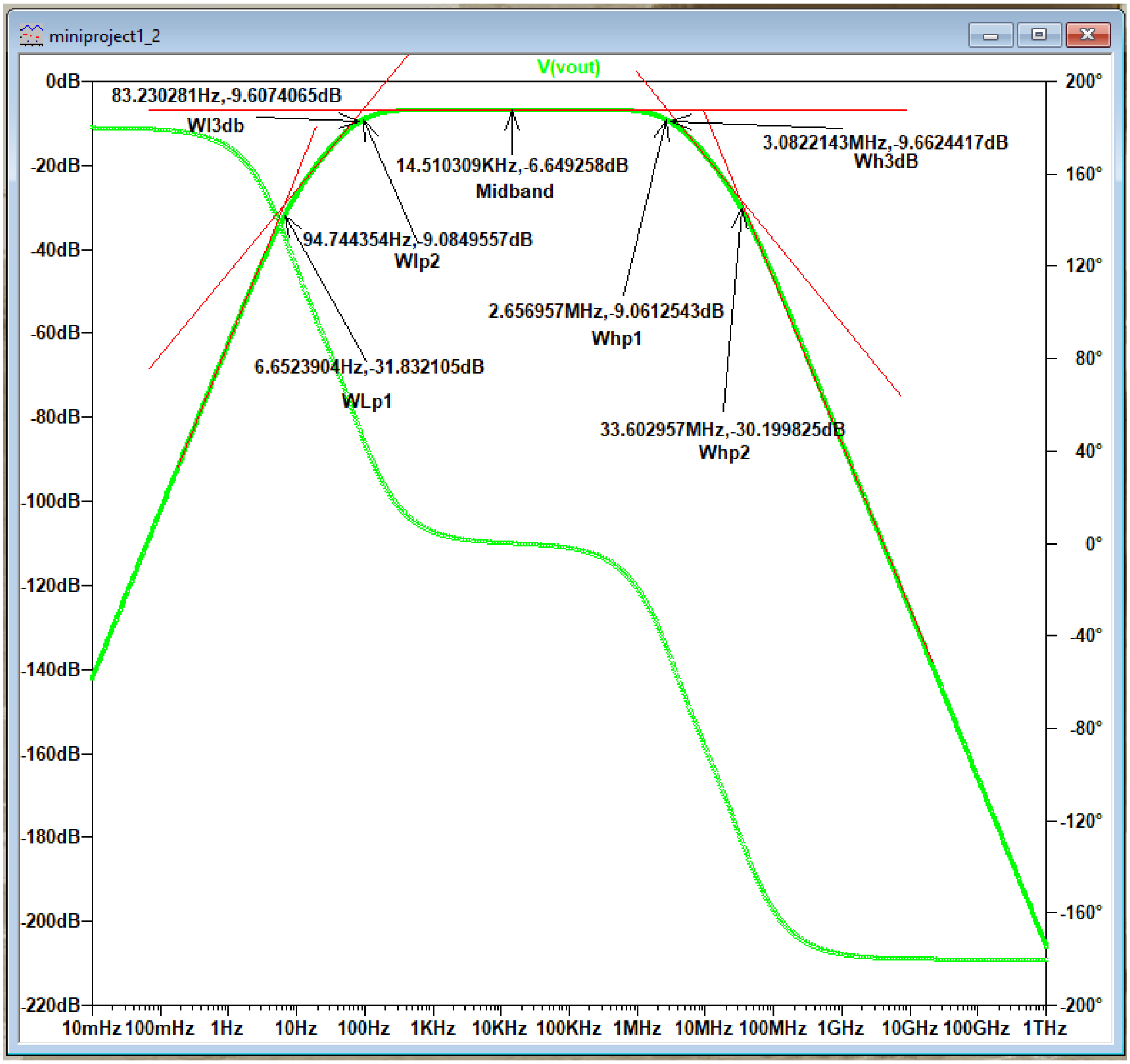

Figure 2.2 Bode plot of the simulated circuit from (figure 2.1) showing mid-band, low and high 3dB values as well as pole locations found graphically

Graphically finding the W_{L3dB} point and comparing it to our calculated value we obtain this table of values

C3	500nF	1µF	2µF	5µF	10µF
W_{L3dB} (Hz)	155.63	78.09	39.60	17.29	12.30
W_{L3dB} (Hz) (graphically)	161.04	83.23	44.05	22.47	15.79
% error	3.476%	6.582%	11%	29.95%	28.37%

As we can see, as the C3 capacitor grows larger so does our percentage of error in our approximation.

The capacitor C3 was responsible for creating the 2nd low frequency pole in most cases, however increasing its capacitance caused the 2nd pole to move to a lower frequency bringing the first 2 low frequency poles closer together. At C3 = 10µF, C3 causes a lower frequency pole than C1. This trend in error shows that approximating the W_{L3dB} frequency using the formula

$$\omega_{L3dB} \approx \sqrt{\omega_{Lp1}^2 + \cdots + \omega_{LpN}^2 - 2\omega_{Lz1}^2 - \cdots - 2\omega_{Lzn}^2}$$

Becomes less accurate the closer the poles are to each other. Which makes sense seeing as when 2 poles get closer together it becomes far more difficult to isolate their individual contributions to the plot.

PART 3:

Given the following circuit

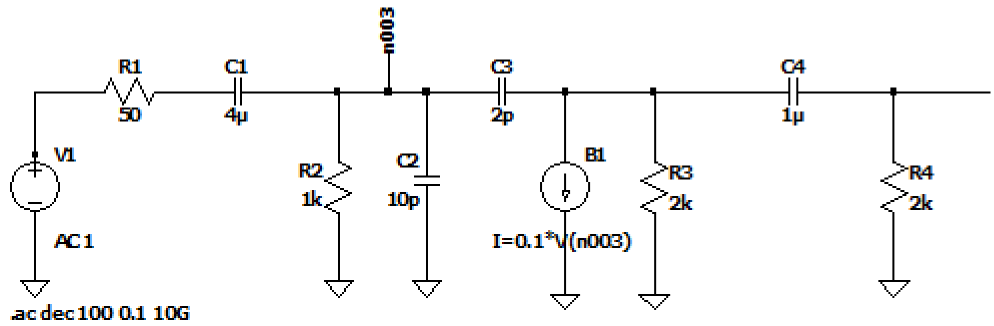

Figure 3.1 The initial circuit of interest

Using Miller's theorem, I arrived at the miller equivalent circuits

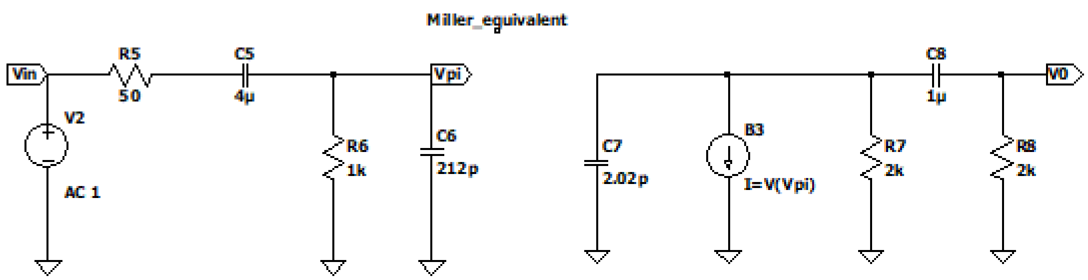

Figure 3.2 The miller approximated circuits of interest

Applying the method of open circuit short circuit time constants, I found the following table of values to describe my transfer function as

$$T(s) = -95.2 * \frac{s}{s+238} * \frac{s}{s+250} * \frac{99*10^6}{s+99*10^6} * \frac{495*10^6}{s+495*10^6}$$

Table of calculated values:

Mid band Gain	-95.2 (V/V)	39.57 (dB)
W_{Lp1}	238 (1/sec)	37.88 (Hz)
W_{Lp2}	250 (1/sec)	39.79 (Hz)
W_{Hp1}	99E6 (1/sec)	1.58E7 (Hz)
W_{Hp2}	495E6 (1/sec)	788E7 (Hz)
W_{L3dB}	345.17 (1/sec)	54.91 (Hz)
W_{H3dB}	1.03E8 (1/sec)	1.64E7 (Hz)

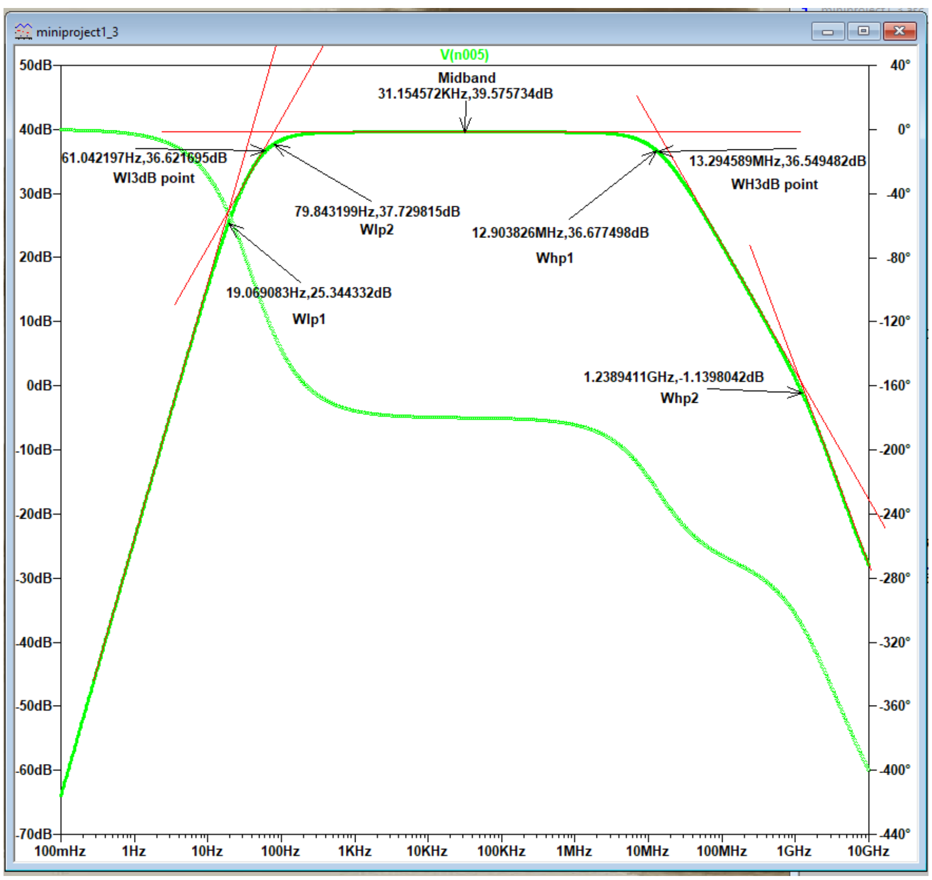

Figure 3.3 The bode plot of my simulated circuit with graphical interpretations of points of interest

I graphically measured

W_{L3dB} to be about 61.04 Hz giving a % error of 11.2% and

W_{H3dB} to be about 13.29MHz giving a % error of 19.0% when compared to my miller theorem calculated values.

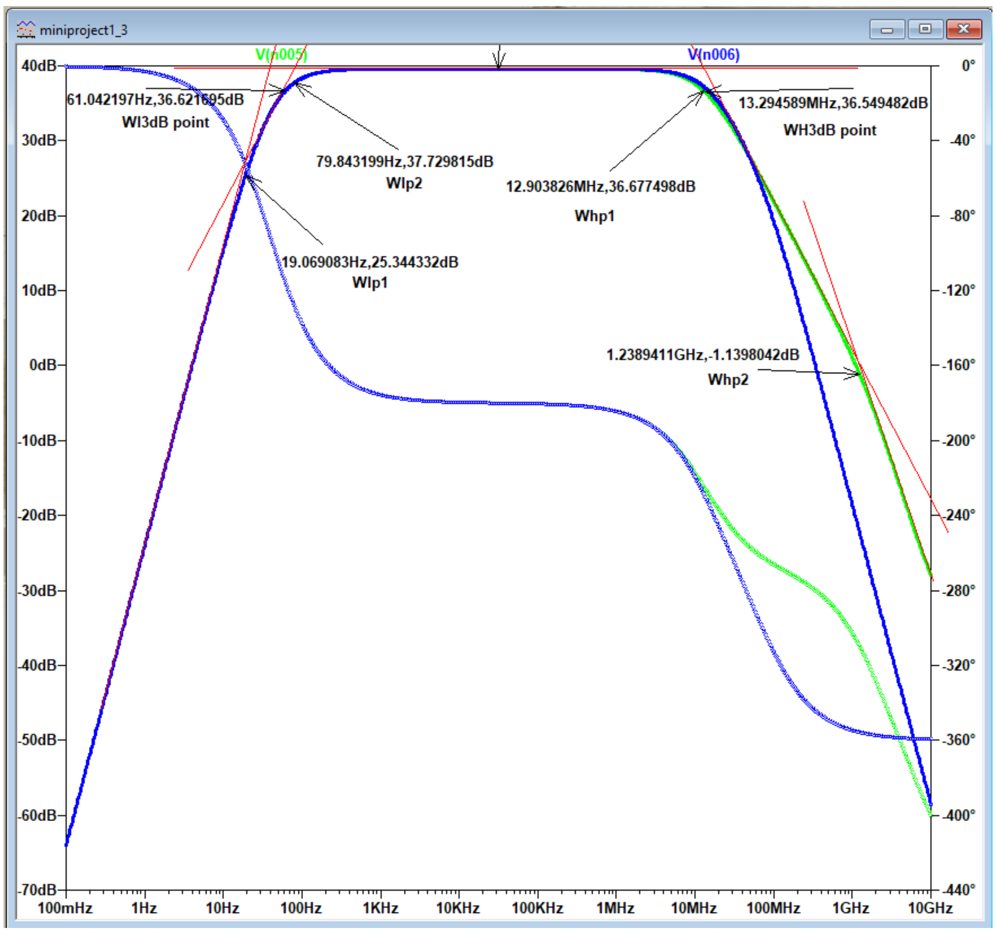

Figure 3.4. A comparison of the simulation of the transfer function calculated by using miller's theorem (blue) and a simulation of the actual circuit (green)

We can see from the above simulation (Figure 3.4) that the miller approximate transfer function is a very accurate depiction of the band pass of the circuit of interest. However, the approximation diverges from the original near the high frequency 3dB point. As shown above (Figure 3.4) there is a zero which contributes a bend to the bode plot of the actual circuit just after the high frequency 3dB point which is neglected by our Miller theorem approximation