Mini-Project 4

UBC Elec 301

Andrew Munro-West 18363572

Table of Contents

Part A An Active Filter

Setup

We are initially given the following Sallen-key configured low pass filter and tasked to find the values of components to turn it into a 2nd order Butterworth filter.

We are given that the transfer function of this filter will be $H(s) = A_M \frac{1/(RC)^2}{s^2 + s * \frac{3 - A_M}{RC} + \frac{1}{(RC)^2}}$

And that $A_M = 1 + \frac{R_2}{R_1}$

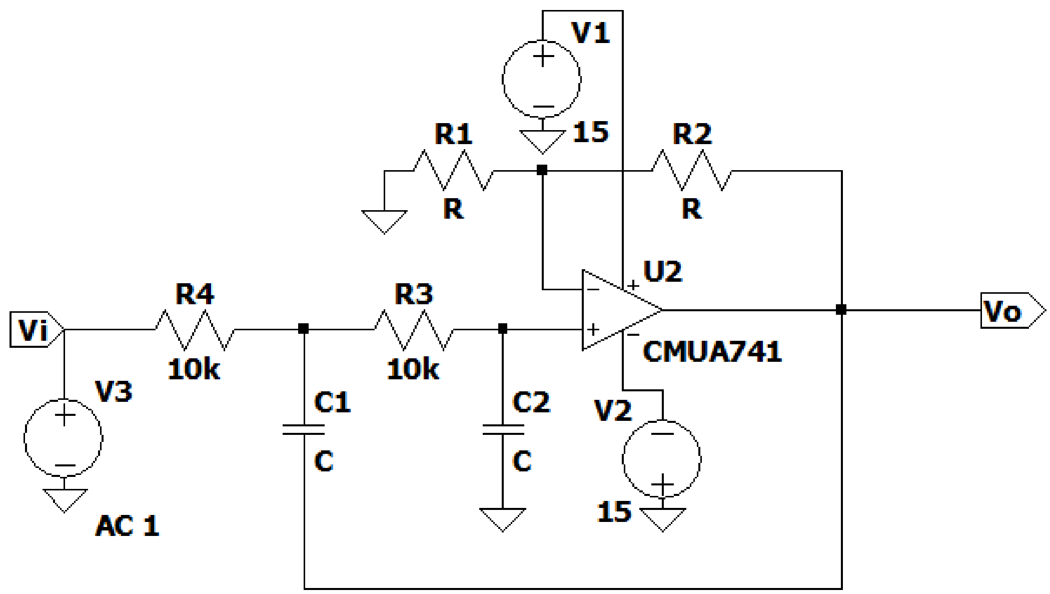


Figure A-1: The initial low pass filter

Part 1 – Capacitor and A_M Values

A Butterworth filter is defined by its complex double poles aligning perfectly spaced on a unit circle in the S plane in order to have pole cancellation across the real axis. As shown by my root locus diagram this means that for a 2nd order Butterworth filter, we must have these poles $45° = \frac{\pi}{2} rad$ from the negative side of the real axis. For higher order Butterworth filters, we change the angle to be have $\frac{\pi}{N} rad$ separation where N is the order of the filter.

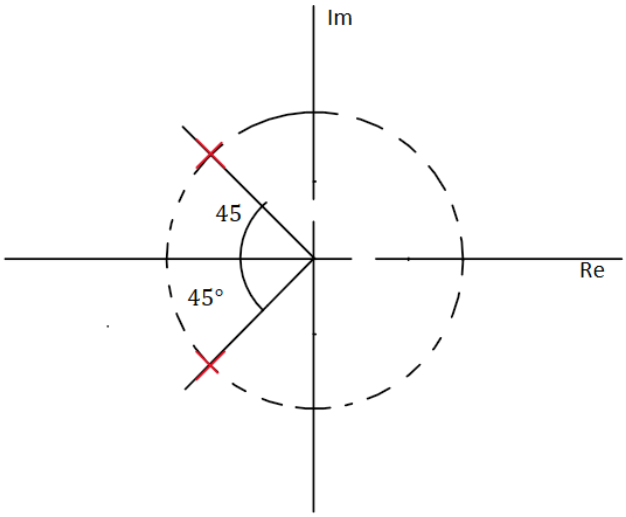


Figure A-2: Root locus of 2nd order Butterworth filter

In order to turn this filter into a Butterworth filter we look at the look up table of normalized Butterworth polynomials and find the form the denominator should take for a 2nd order Butterworth filter. For a second order butter worth this should be: $(s^2 + 1.414s + 1)$

The denominator of our filter can be rewritten as $s^2 + 2\zeta\omega_c s + \omega_c^2$

Where the cut frequency $\omega_c = \frac{1}{RC}$, and the damping factor $\zeta = \frac{3-A_M}{2}$

Using the fact that we want a cut frequency of 10kHz and that we are given R as 10kΩ we can solve for our C. $\omega_c = \frac{1}{10000*C} = 2\pi * 10000 \therefore C = 1.6nF$

To make a 2nd order Butterworth filter we need

$$2\zeta = \sqrt{2} = (3 - A_M) \therefore A_M = 1.5857$$

$$A_M = 1 + \frac{R_2}{R_1} = 1.5857 \therefore \frac{R_2}{R_1} = 0.5857$$

And since we are told $R_2 + R_1 = 10000$ we find $R_1 = 6307.89,\ R_2 = 3692.106$

Plugging in values and simulating we get the following Bode Plots

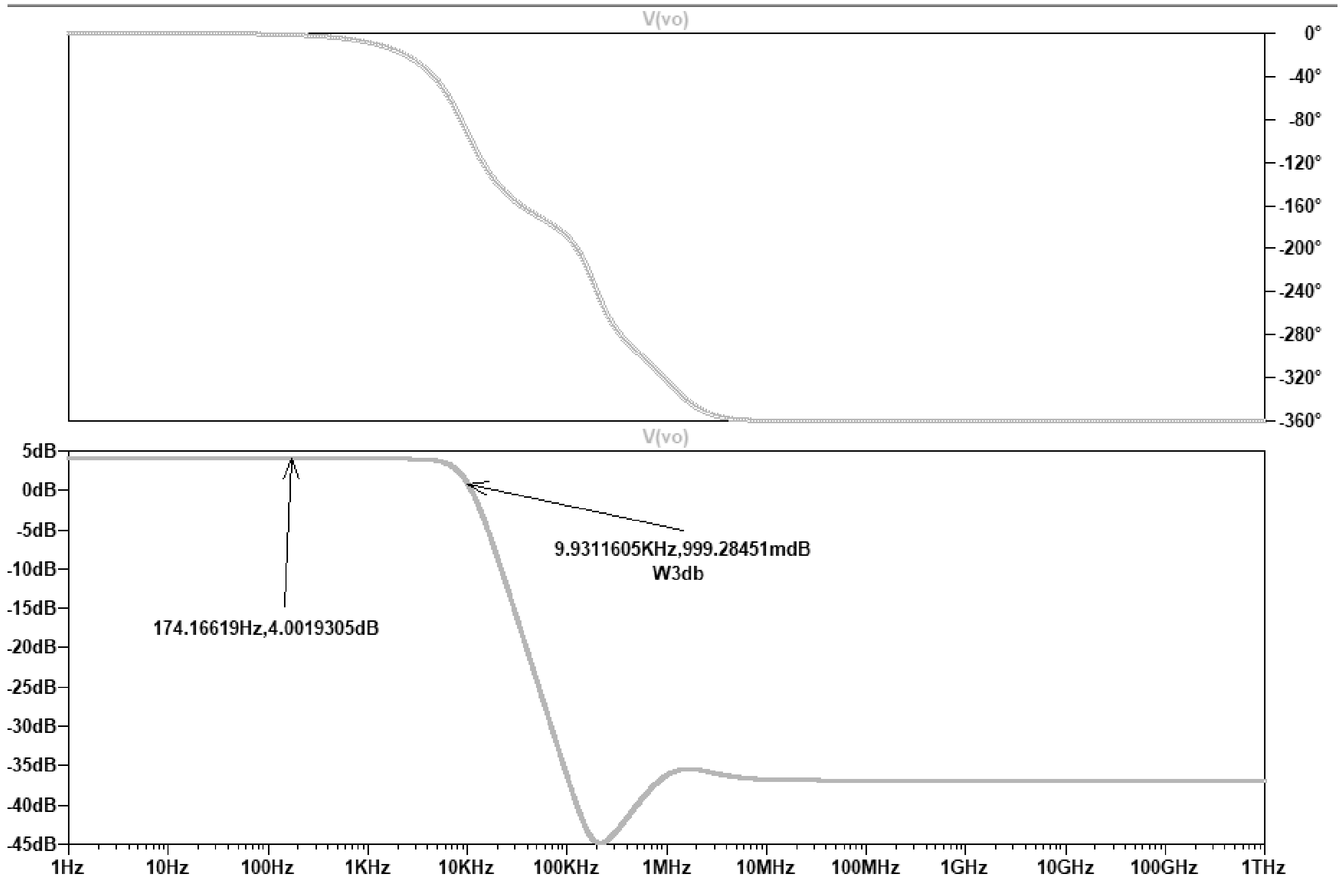


Figure A-3: Bode plot of 2nd order Butterworth filter magnitude(bottom) and phase (top)

Part 2 – Oscillation

By removing the source and grounding the input of our circuit I began running tests to determine at what A_M value the circuit will begin to start oscillating. In order to do this, I set step functions to change the ratio or my resistors R1 and R2. I started by stepping through values of R1 =1k to 9k (and the inverse for R2) in 1k increments while observing the behaviour. Then noting the point of extreme change, I changed the limits of my step and refined my search. (additional plots can be found in appendix)

In the end I found the point where my circuit begins oscillating to be at $R1 = 3331\Omega$ and $R2 = 6669\ \Omega$ these resistor values correspond to $A_M = 3$ which makes sense as when $A_M = 3$ the second term of our denominators polynomial goes to 0 and we get $s^2 + \omega_c^2$ as the denominator of our transfer function (complex conjugate poles at 0). This transfer function results in the following root locus.

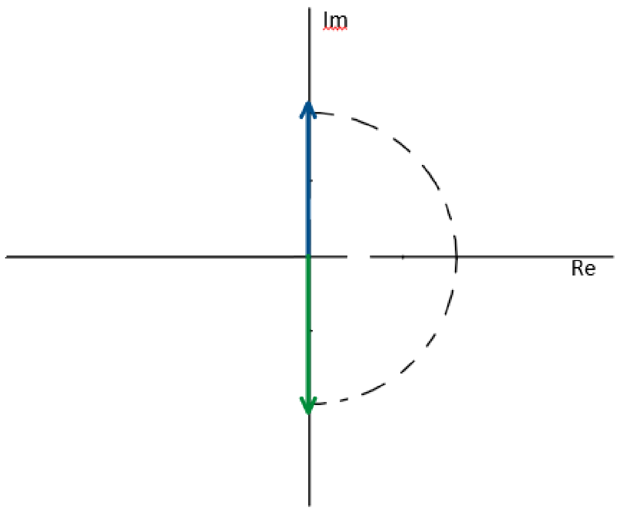


Figure A-4: root locus of point where circuit begins oscillating

As can be seen by the root locus, our poles now both lands right on the imaginary axis with a real component of 0. At this point the circuit will have an infinite output for zero input signal at the frequency ω_c. Poles on the right side of the imaginary axis are unstable, as we approach the right side of the imaginary axis, we see that our circuit begins to oscillate. This oscillation is due to the feedback loop in our circuit. When the poles of our transfer function start getting to the right side of the imaginary axis our circuit begins to amplify its output, with the feedback loop our circuit begins amplifying the output exponentially until we reach an upper limit. At the upper limit our gain stops increasing and we see the following oscillation with a frequency equal to our ω_c.

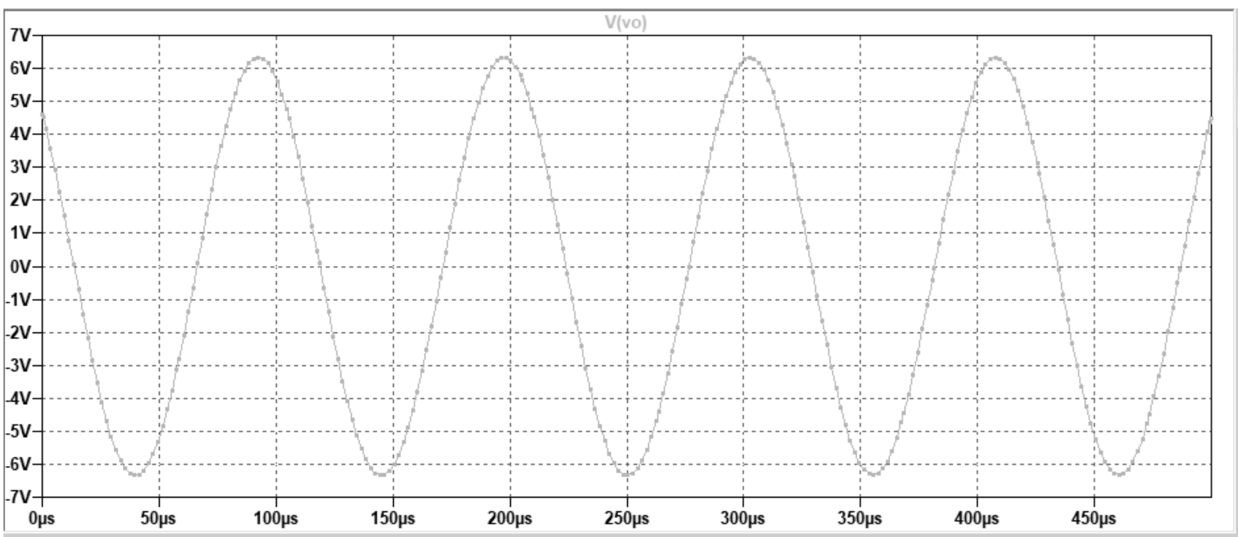


Figure A-5: oscillation waveform

As can be seen in figure A-5 our waveform is oscillating at the expected frequency of 10kHz the same as our cut frequency ω_c.

Part B A Phase Shift Oscillator

Setup

We are initially given the following phase shift oscillator to observe.

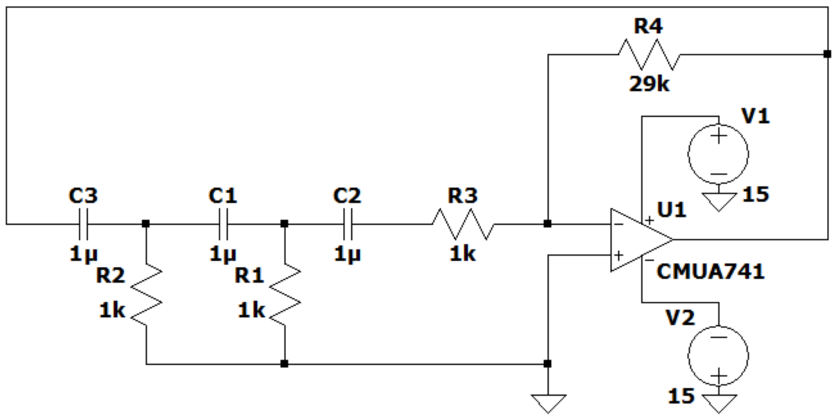


Figure B-1: Phase shift oscillator circuit

Part 1 – Analysis

Given the circuit in figure B-1 and changing the 29k to 29.1k we manage to create an oscillating waveform given no input signal. Through experimentation I found that you could add as little as 10Ω to obtain oscillation however it would lake significantly longer to reach its maximum oscillation amplitude.

The way that this oscillator works is through an op-Amp feedback network. the transfer function of such a configuration should look like: $A_f(j\omega) = \frac{A(j\omega)}{1+A(j\omega)*\beta(j\omega)}$ in order to have oscillation we will require poles on the $j\omega - axis$. This means that

$1 + A(j\omega) * \beta(j\omega) = s^2 + \omega_o^2$ this will give us a frequency ω_o where the gain of the loop

$A\beta = -1,\ A_f$ becomes infinite at this frequency. At ω_o the phase of the loop gain needs to be $-180°$ and the magnitude unity. This is called the “Barkhausen criteria”.

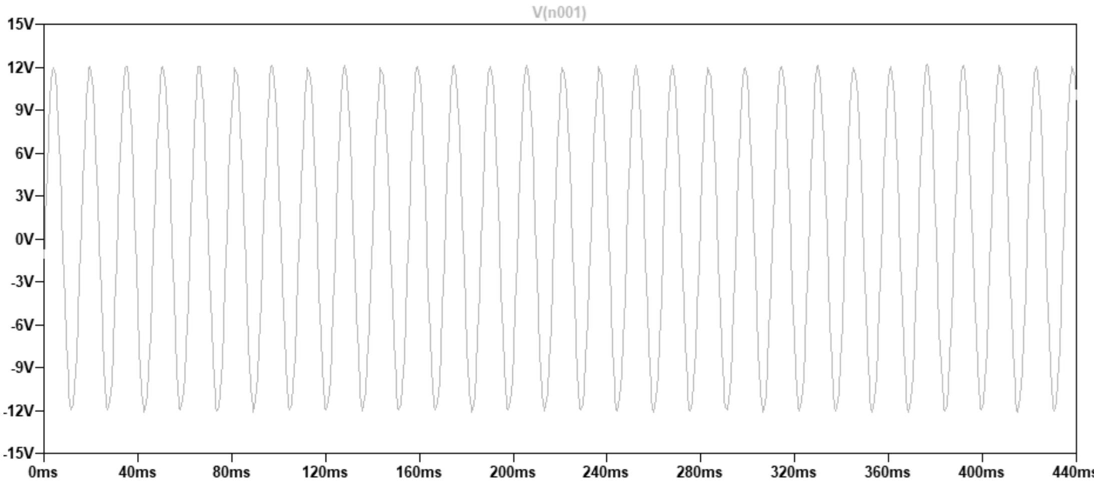


Figure B-2: Oscillation waveform

As seen in figure B-2 the circuit oscillates at a frequency of 62.5 Hz

It was proven in the appendix of our lecture notes that for this specific configuration of phase shift oscillator the oscillation frequency $\omega = \frac{1}{\sqrt{6}*RC}$ and $k = -29$ as a minimum gain between output and input is necessary for our phase shift oscillator to oscillate.

Using the equation for frequency we can quickly compute the expected value of frequency after changing our circuit. By changing the values of all my resistors and all my capacitors by factor of 2 and ½ we can see that frequency drops as RC increase which is expected according to our equation.

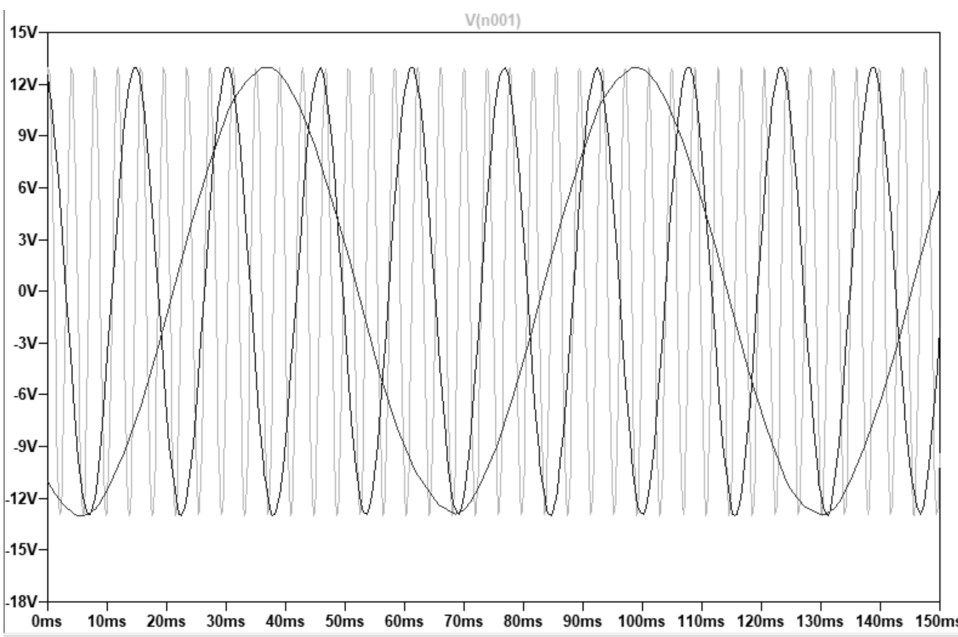


Figure B-3: All 3 waveforms of interest double(largest wavelength), Halved (shortest wavelength)

As shown in figure B-3 we can see the following for the frequency of oscillation:

Circuit	R (Ω)	C (F)	Measured f (Hz)	Calculated f (Hz)	Error
Halved	500	0.5μ	250	259.89	3.80546%
Regular	1k	1μ	66.7	64.97	2.66277%
Doubled	2k	2μ	15.8	16.24	2.70936%

As you can see our equation for the frequency of oscillation is very accurate, providing less than 4% error for all our measurements.

Part C A Feedback Network

Setup

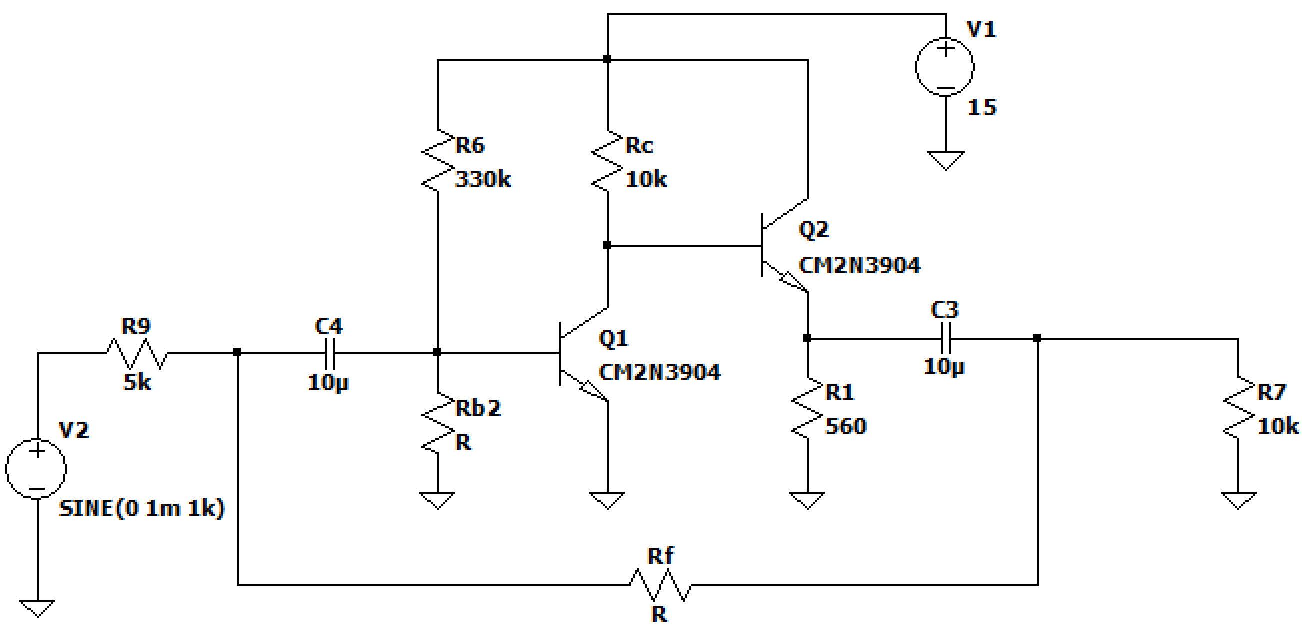


Figure C-1: The Circuit of interest

By opening the feedback resistor R_f, applying a 1mV 1kHz input signal and then varying the value of R_{B2} from 1k to 100k I found that the Maximum open loop gain of our circuit can be found with $R_{B2} = 20.1k\Omega \approx 20k$ at which point we have a gain of -127 V/V

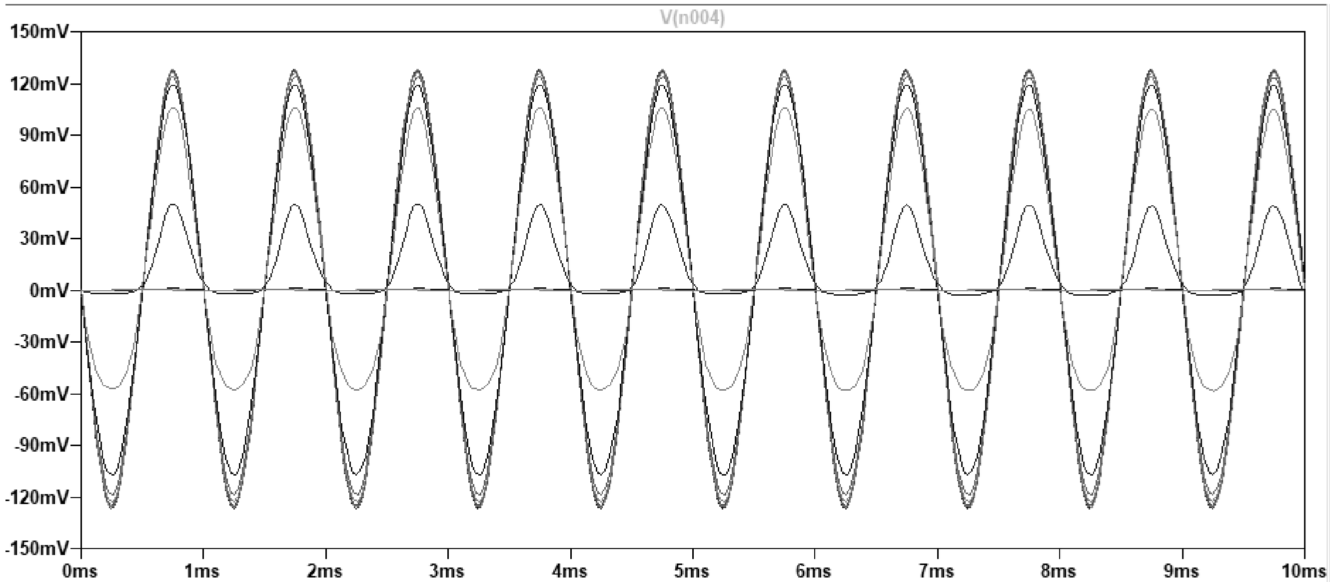


Figure C-2: output waveform for Rb2 from 20k to 22k

Part 1 – The DC operating point parameters

Running the dc operating point analysis, I obtained the following values directly from my spice software:

Name:	Model:	Ib:	Ic:	Vbe:	Vbc:	Vce:	BetaDC:	Gm:
q2	cm2n3904	1.54E-05	2.19E-03	6.65E-01	-1.31E+01	1.38E+01	1.42E+02	7.90E-02
q1	cm2n3904	1.08E-05	1.29E-03	6.54E-01	-1.25E+00	1.90E+00	1.20E+02	4.77E-02
Rpi:	Rx:	Ro:	Cbe:	Cbc:	Cjs:	BetaAC:	Cbx:	Ft:
2.06E+03	0.00E+00	5.16E+04	3.87E-11	1.34E-12	0.00E+00	1.62E+02	0.00E+00	3.14E+08
2.98E+03	0.00E+00	7.82E+04	2.61E-11	2.53E-12	0.00E+00	1.42E+02	0.00E+00	2.65E+08

Specifically, we are asked for h_{fe} g_m and r_π which correspond to

	h_{fe}	g_m	r_π
Q1	120	0.0477	$2.98k\Omega$
Q2	142	0.0790	$2.06k\Omega$

Part 2 – Measured open loop and predicted closed loop

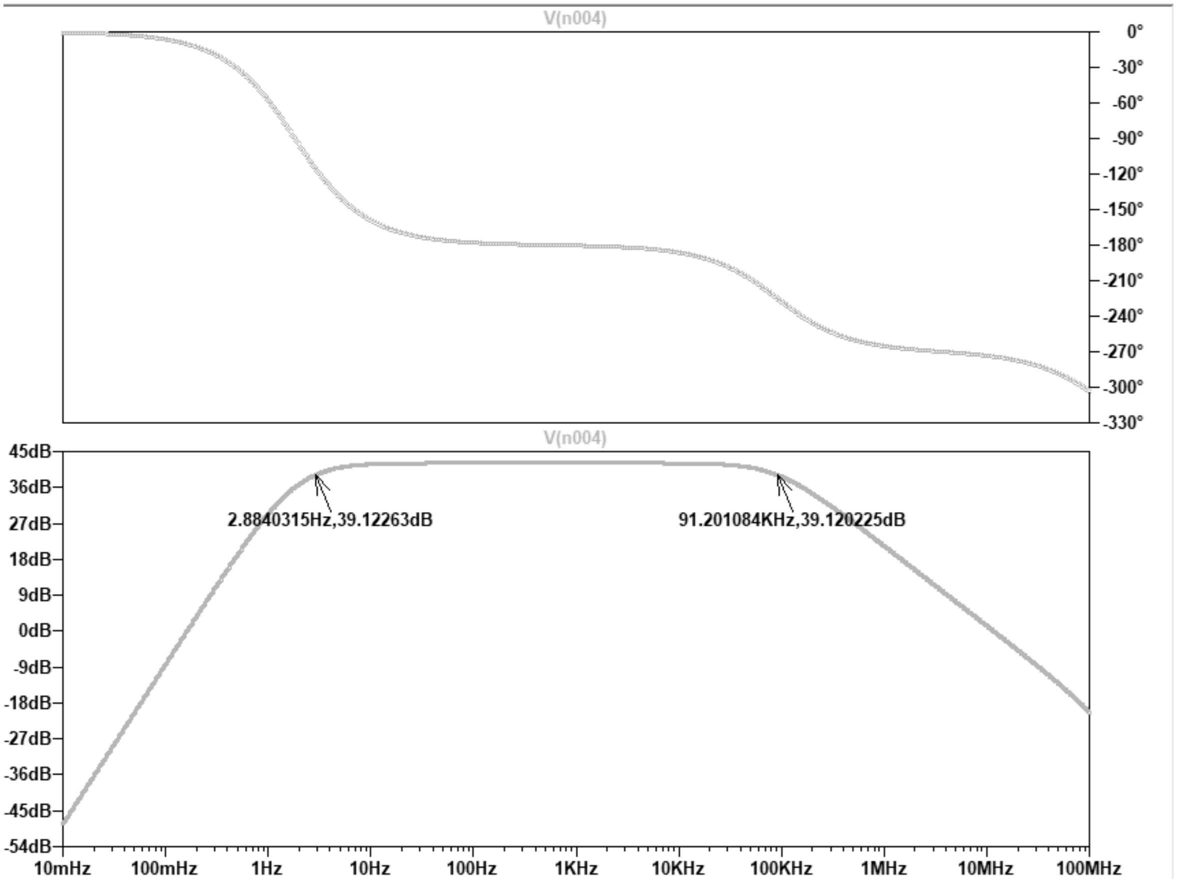


Figure C-3: Bode plot Magnitude(Bottom) , Phase (top)

As shown in figure C-3 the low 3dB cut is at 2.88Hz the high 3dB cut is at 91.2kHz and we have a midband gain of about 42.1 dB = |127| V/V as we expected.

Applying a test source to measure input and output impedance we find:

R_{in}	R_{out}	$\lvert A_m \rvert$	ω_{L3dB}	ω_{H3dB}
$2.6k\Omega$	58.8Ω	127 V/V	2.88Hz	91.2kHz

Now we need to find the predicted closed-loop frequency response and both input and output resistance at 1khz.

In order to predict the closed loop behaviour, we need to know the feedback topology and look at the appropriate parameter network. By looking at the circuit we can determine that the feedback topology is shunt-shunt, the output voltage is being sampled and then current is being mixed into the input to create our feedback network. Another way to look at it would be that the feedback resistor connects the input and the output in a parallel connection on both sides. The appropriate parameter for shunt-shunt topologies is the y-parameter network:

$$I_1 = y_{11}V_1 + y_{12}V_2$$

$$I_2 = y_{21}V_1 + y_{22}V_2$$

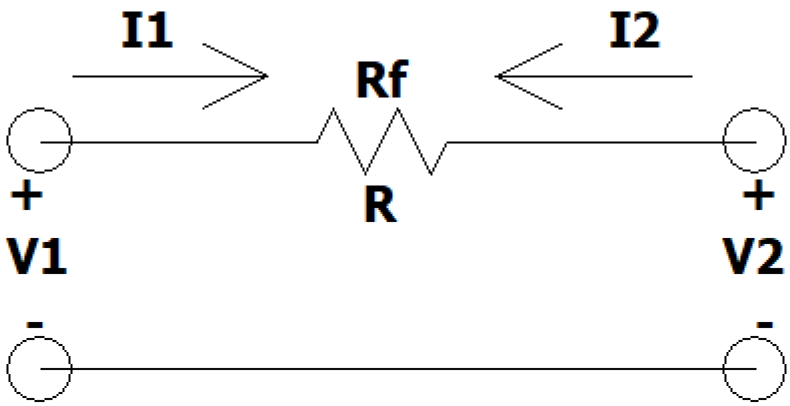


Figure C-4:The Feedback Network

We compute the y parameter matrix for useful parameters, neglecting the feedforward of the feedback network y_{21}

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} = \frac{1}{R_f}, \qquad y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} = -\frac{1}{R_f} = \beta\ ,$$

$$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} = \frac{1}{R_f}$$

Given that $R_f = 100k\Omega$, we have a feedback gain of $\beta = -10\mu S = y_{12}$

Since our feedback topology is shunt-shunt (current in - voltage out), in order to calculate the closed loop gain of our amplifier, we need our open loop gain A in terms of V/A:

$$A' = \frac{V_o}{I_s} = \frac{V_o}{\frac{V_s}{R_s}} = R_s * \frac{V_o}{V_s} = R_s * A = 5k * -127 = -635\ kV/A$$

This works because R_s Is the impedance of the source, if we were to treat our input source as a current source it should supply a current of V_s/R_s

We can then plug values into our feedback equation $A_f = \frac{A'}{1+\beta A'} = \frac{635k}{1+10\mu*635k} = 86.4kV/A$ dividing by the source impedance to return back to V/V we get feedback gain of 17.2V/V.

By applying feedback, we also extend the bandwidth. As shown in class as feedback is applied the midband gain drops and the cut frequency points move farther away from each other extending the bandwidth by a factor of $1 + A'\beta = 7.35$ on each side

$$f_{H3dBf} = f_{H3dB}(1 + A'\beta) = 91.2k * (1 + 635k * 10\mu) = 670.3kHz$$

$$f_{L3dBf} = \frac{f_{L3dB}}{1 + A'\beta} = \frac{2.88}{7.35} = 0.391Hz$$

Finally, we look at the input and output impedance. For the shunt-shunt feedback amplifier we have a Trans-resistance amplifier, A is a trans-resistance and β is a transconductance. The negative feedback will decrease both the input and output impedances by the amount of feedback, $R_{if} = \frac{R_i}{1+A'\beta} = \frac{2.6k}{7.35} = 353.7\Omega$, $R_{of} = \frac{R_o}{1+A'\beta} = \frac{58.8}{7.35} = 8\Omega$

To summarize our predicted closed loop response:

	R_{if}	R_{of}	|Am|	ω_{L3dB}	ω_{H3dB}
calculated	353.7Ω	8Ω	17.2 V/V	391mHz	670.3kHz
Measured	242.7Ω	7.87Ω	17.24V/V	513mHz	676.1kHz

Our calculated values agree with our measured values with good accuracy. Its worth noting that the measured ω_{L3dB} is slightly larger but still within the same magnitude. This is most likely caused by the sudden increase in gain towards the left side of our midband. We also see that our calculated input impedance is less accurate than our calculated output impedance.

Part 3 – Measured closed loop response and β's for various R_f

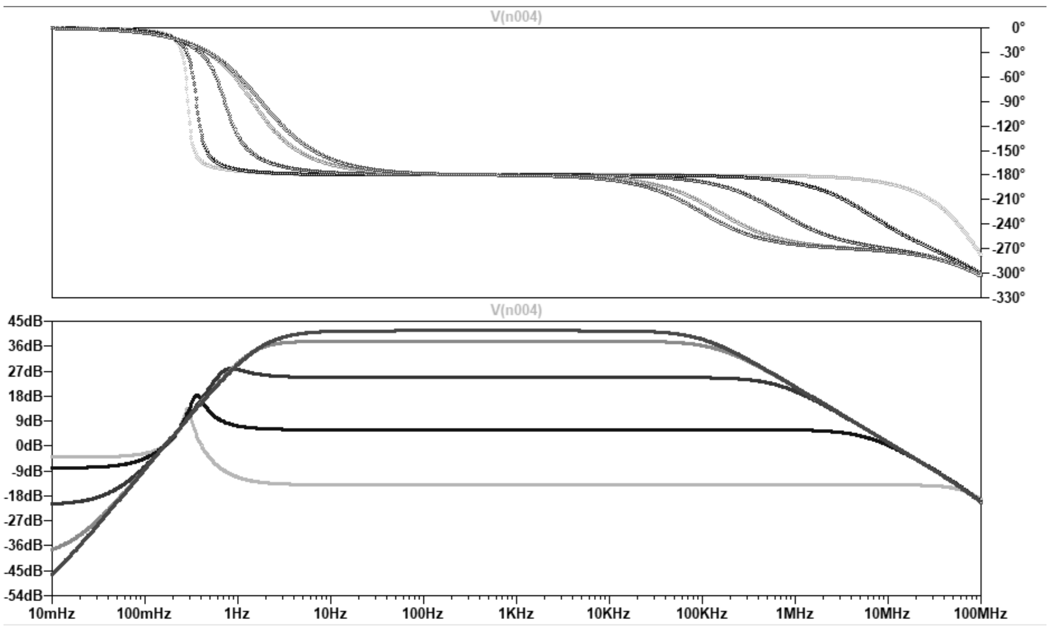


Figure C-5: Closed loop frequency response magnitude(bottom) / Phase(top) for

Rf = 1k (lowest line) 10k 100k 1M 10M (Highest line)

Using the value $A' = 635\ kV/A$ found in the previous part we can measure the closed loop midband gain, convert it to V/A by multiplying by the source impedance and use the relationship $A_f = \frac{A'}{1+\beta A'}$ to find βcomparing these values to our calculated β of $-\frac{1}{R_f}$ we get:

R_f	A_f (V/A)	β Measured(S)	β Calculated(S)
1k	-995	-1.0034m	-1m
10k	-9800	-0.1004m	-0.1m
100k	-86.2 k	-10.026μ	-10μ
1M	-389 k	-0.996μ	-1μ
10M	-599.45 k	-0.093μ	-0.1μ

We can see that our calculated β values are very close to the measured.

We also observe that as R_f drops to lower values we see a spike in our amplitude plot on the low frequency side of our midband. I believe the reason for this sudden spike at low frequencies is due to the coupling capacitor on the output side of our circuit. As we can see from the phase plot of our circuit there is a $180\ °$ phase shift in our circuit that coincides with our spike in amplitude. It can be observed that as $R_f$decreases in magnitude the $180°$ phase shift gets a steeper slope. The 2 larger $10\mu F$ capacitors are responsible for the low frequency poles and the output capacitor seems to have an effect in relation to the feedback resistors size. A magnitude plot of my circuit using $R_f = 10k$ which varies the value of the output coupling capacitor can be found in the appendix which seems to support my idea that that capacitors relationship with R_f is the cause of this behaviour.

Part 4 – Amount of Feedback from input and output resistance

First, we set up our circuit and measure the closed loop I/O resistances for our 3 values of R_f.

R_f	R_{if}	R_{of}
10k	26.5	1.01
100k	242	7.8
1M	1315.8	34.97

We are told to compare the estimated amount of feedback with the predicted. The “amount of feedback” is referring to $1 + A'\beta$. To estimate the amount of feedback we use the relationship between the open and closed loop input/output impedances. $R_{if} = \frac{R_i}{1+A'\beta}$, $R_{of} = \frac{R_o}{1+A'\beta}$ inputting our measured values and solving for $1 + A'\beta$. For our predicted values we use our value of $A'\ and\ \beta$ from the previous parts. $A' = -635\ kV/A$, and for β, the corresponding measured values from part 3.

Using $R_i = 2.6k\Omega$, $R_o = 58.8\Omega$ from our open loop measurements:

R_f	R_i/R_{if}	R_o/R_{of}	Estimated Feedback	Predicted Feedback
10k	98.11	58.22	78.17	64.754
100k	10.74	5.47	8.105	7.3665
1M	1.976	1.681	1.8285	1.6325

This data shows us that using the relationship between the open and closed loop I/O resistance is a decently accurate method of estimating the amount of feedback in our circuit. We also see that the amount of feedback in our circuit will drop as the feedback resistor in our system increases. This makes sense as we are looking at a shunt-shunt feedback network meaning we are mixing current into our input to cause feedback, so increasing the size of our feedback resistor will reduce the amount of current being fed back into our input and reduce the amount of feedback.

Part 5 – The De-sensitivity Factor

For the De-sensitivity factor, we want to see how much the feedback gain changes with the open loop gain. Differentiating $\frac{dA_f}{dA} = \frac{1}{1+A\beta} - \frac{A\beta}{(1+A\beta)^2} = \frac{1}{(1+A\beta)^2}, \frac{dA_f}{A_f} = \frac{1}{1+A\beta}\frac{dA}{A}$ as we see here the closed loop gain changes with the open loop gain after being De-sensitised by a factor of $1 + A\beta$. For the expected value we use the previously found $\beta = -0.01\ mS$ and $A' = 635k$ for a 100k feedback resistor giving us a de-sensitivity factor of $1 + -635k * -10.026\mu = 7.3665$ for $R_f = 100k$

R_c	AM open loop (V/A)	AM closed loop $R_f = 100k$ (V/A)	Observed
9.9k	-634.2k	-86.17k	7.360
10k	-637.75k	-86.24k	7.395
10.1k	-640.99k	-86.30k	7.42

As we can see from the data it is apparent that the de-sensitivity factor will increase as the gain of our amplifier increases.

Conclusion:

Throughout this mini project we learned about feedback in several different capacities.

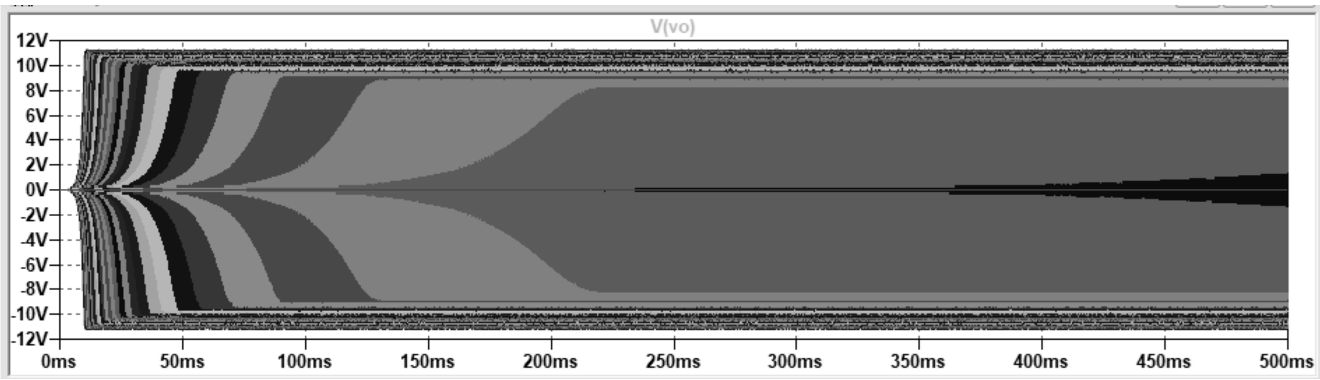
First, we built a low pass Butterworth filter. In designing the filter, we learned about the affects of complex conjugate poles in the s-plane. We then converted our Butterworth filter into an oscillator by moving its complex poles onto the jw-axis and then slightly into the right side of the jw-axis.

In the second part we built a phase shift oscillator. The phase shift oscillator works on a similar principle to the oscillator we built in the first part; the oscillation is caused by complex conjugate poles on the jw – axis. The feedback loop causes the unstable poles to increase the input with every iteration of the loop, depending on how far to the right of the axis the poles are the gain will change and it will take longer for the circuit to oscillate at its maximum value.

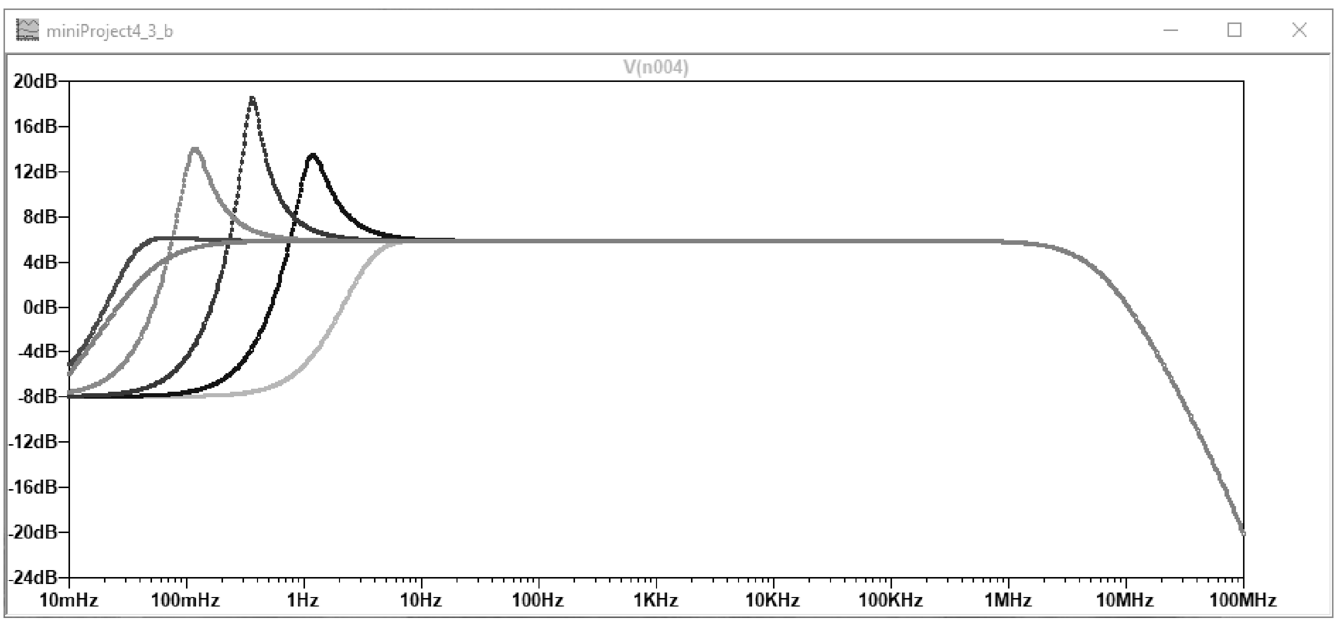
In the third and final part we analyzed a feedback network. Identifying the feedback network as a shunt-shunt topology we went through the process of calculating, measuring and comparing the open and closed loop responses of the feedback amplifier in order to better understand the behavior and influence of feedback in our circuits. We calculated both the amount of feedback in our network as well as the de-sensitivity factor and observed how they affect the gain, midband and I/O resistances of our circuit.

Through this project we've learned and gained a fair bit of experience working with oscillators and feedback networks. A very useful takeaway from this project has been my newfound understanding of feedback topologies.

Appendix:



Part A -1 Oscillation behavior as A goes from 10 to 0 visually describing the effect of pole position on oscillation behavior.



Part C-5 amplitude response for values of C_{C2} from 0.1u to 10m showing the effect of the steeper 180° phase shift caused by the interaction of $R_f\ and\ C_{C2}$